South East Asian J. of Mathematics and Mathematical Sciences
Vol. 20, No. 2 (2024), pp. 53-66

DOI: 10.56827/SEAJMMS.2024.2002.5 **ISSN (Online): 2582-0850**

ISSN (Print): 0972-7752

CONJUGACY CLASS GRAPHS OF SOME K-METACYCLIC GROUPS

Yangertola and Kuntala Patra

Department of Mathematics,
Gauhati University,
Guwahati, Assam - 781014, INDIA

E-mail : yangertola17@gmail.com, kuntalapatra@gmail.com

(**Received:** *Jun. 11, 2024* **Accepted:** *Aug. 10, 2024* **Published:** *Aug. 30, 2024)*

Abstract: A class of K-metacyclic group of order $p(p-1)$ denoted by G, has group presentation $x^p = y^{p-1} = 1$; $y^{-1}xy = x^r$; $(r-1,p) = 1$ where p is an odd prime and r is a primitive root modulo p. To this group, we attach a simple undirected graph Γ_G^{cc} whose vertices are the conjugacy classes of G and two distinct vertices x and y are connected by an edge if the *gcd* of the class size of x and y is greater than 1. In this paper, Γ_G^{cc} and $\Gamma_{G\times G}^{cc}$ are obtained and then different graph theoretic properties like planarity, clique number, chromatic number, independence number, clique polynomial, independence polynomial, dominating number, spectrum and energy of these graphs are studied. The line graph of Γ_G^{cc} is found to be a regular graph and the complement graph of Γ_G^{cc} is found to be a star graph. Various aspects of the line graph and the complement graph are also determined in this paper.

Keywords and Phrases: Conjugacy class graph, K-metacyclic group, line graph, complement graph.

2020 Mathematics Subject Classification: 05C25, 05C69, 05C76.

1. Introduction

In the study of group theory, various types of graphs such as Cayley graphs, power graphs, commuting graphs etc have been used to investigate the structural properties of groups. Recently, constructing graphs based on the conjugacy classes of groups has become a significant area of research. Among these are the conjugate graphs, conjugacy class graphs, and generalized conjugacy class graphs. In this context, we focus on the conjugacy class graphs of K-metacyclic groups, which provide a distinctive approach in understanding the conjugacy relationships within the groups.

Given a finite group G and two elements g and h in G, the elements g and h are said to be conjugate if there exists some x in G such that $x^{-1}gx = h$. This operation is known as conjugation, and the set of all the elements conjugate to g is called the conjugacy class of g. Conjugacy classes offer a deep understanding of the internal structure and properties of groups, which is why it has been a key area of research for long time. The conjugacy class graph Γ_G^{cc}, is a graph whose vertices are the conjugacy classes of G and two distinct vertices x and y are connected by an edge if the class size of x and y have a *gcd* greater than 1. This graph was first introduced by Bertram et al [2] in 1990, and has hence been studied extensively over the years. In 2012, Bianchi et al [3] proved that the conjugacy class graph of a finite group G is a 2-regular graph if and only if it is a complete graph of order three and a 3-regular graph if and only if it is a complete graph of order four. Moradipour et al (2013) [15] determined the conjugacy class graphs of some metacyclic 2-groups and then studied different properties like clique number, chromatic number etc. Khoshnevis et al (2015) [12] computed the conjugacy class graphs of Special linear groups and also studied its various properties. Sarmin et al (2016) [18] obtained the conjugate graphs and conjugacy class graphs by computing the conjugacy classes of metacyclic 2-groups of order at most 32. Najmuddin et al (2018) [16] determined the independence and clique polynomial of the conjugacy class graph of dihedral group of order 2n. Zulkarnain et al (2020) [20] obtained the conjugacy class graphs for some non abelian 3-groups. Motivated by these studies, we aim to obtain the conjugacy class graphs of K-metacyclic group G and its direct product $G \times G$. We also study different graphical parameters of the resultant graphs.

2. Preliminaries

This section provides a comprehensive list of definitions and results of group theory and graph theory which will be used throughout this paper.

Definition 2.1. [17] *A K-metacyclic group G is a group of order* $p(p-1)$ *generated by the elements* x *and* y *with defining relations:* $x^p = y^{p-1} = 1$; $y^{-1}xy = x^r$; $(r-1, p) = 1$, *where* r *is a primitive root modulo* p.

Definition 2.2. [9] *An integer* b *is a primitive root modulo* m *if* b *is coprime to* m *and the order of* $b(mod\ m)$ *is* $\phi(m)$.

Definition 2.3. [7] *The chromatic number* $\chi(X)$ *of a graph* X *is defined as the* *smallest number of colours needed to colour its vertices such that no two adjacent vertices share the same colour.*

Definition 2.4. [7] *A clique is defined as a subset C of vertices in a graph X, where the induced subgraph on C forms a complete graph. The clique number of the graph X denoted by $\omega(X)$ represents the maximum size of a clique.*

Definition 2.5. [10] *The clique polynomial of a graph X is the polynomial whose coefficient on x^k is given by the number of cliques of order k in X. This is denoted by $C(X;x)$. So $C(X;x) = \sum_{k=0}^{\omega(X)} c_k x^k$ where c_k is the number of cliques of order k and $\omega(X)$ is the clique number of X.*

Definition 2.6. [7] *An independent set in a graph X is defined as a subset Y of its vertices, where the induced subgraph on Y does not contain any edges. The independence number of the graph X, denoted by $\alpha(X)$ represents the maximum size of an independent set in X.*

Definition 2.7. [16] *The independence polynomial of a graph X is the polynomial whose coefficient on x^k is given by the number of independent sets of order k in X. This is denoted by $I(X;x)$. So, $I(X;x) = \sum_{k=0}^{\alpha(X)} \alpha_k x^k$ where a_k is the number of independent sets of size k and $\alpha(X)$ is the independence number of X.*

Definition 2.8. [19] *Consider a subset S of the vertices of a graph Γ, and represent the set of vertices in Γ, that are either in S or adjacent to a vertex in S as $N_\Gamma[S]$. If $N_\Gamma[S] = V(\Gamma)$, then S is referred to as a dominating set for Γ. The dominating number $\gamma(\Gamma)$ is the smallest size of a dominating set for the vertices of Γ.*

Definition 2.9. [7] *The spectrum of a matrix is the list of its eigenvalues together with their multiplicities.*

Theorem 2.10. [19] *A K-metacyclic group G of order $p(p-1)$ has $p-1$ non central conjugacy classes and they are:*
$cl(x) = \{x, x^2, x^3, \ldots, x^{p-1}\}$,
$cl(y) = \{y, yx, yx^2, yx^3, \ldots, yx^{p-1}\}$,
$cl(y^2) = \{y^2, y^2x, y^2x^2, y^2x^3, ..., y^2x^{p-1}\}$,
$\vdots$
$cl(y^{p-2}) = \{y^{p-2}, y^{p-2}x, y^{p-2}x^2, y^{p-2}x^3, ..., y^{p-2}x^{p-1}\}$.

3 Main Results

The present section is structured into three subsections. In the first subsection, we derive the conjugacy class graph Γ_G^{cc} subsequently delving into diverse graph properties, including planarity, clique number, clique polynomial, chromatic number, dominating number, independence number, and independence polynomial.

Following this, the second subsection pertains to the construction and analysis of the conjugacy class graph of $G \times G$ with a focus on unveiling its various properties. Finally, the third subsection focuses on the line graph and complement graph of the conjugacy class graph of a K-metacyclic group.

3.1. Conjugacy class graph of a K-metacyclic group

A conjugacy class graph, denoted by Γ_G^{cc}, is a graph whose vertices are the conjugacy classes of a group G and two distinct vertices x and y are connected by an edge if the class size of x and y have a *gcd* greater than 1. If G represents the conjugacy class graph of a K-metacyclic group of order $p(p-1)$, we obtain the following results:

Theorem 3.1.1. *Γ_G^{cc}, consists of a union of one singleton graph and one complete graph of order $p-2$ with $\frac{(p-2)(p-3)}{2}$ edges.*
Proof. Since there are $p-1$ non central conjugacy classes in G, Γ_G^{cc} will have $p-1$ vertices. Also, only one of those conjugacy classes is of cardinality $p-1$ and the rest of the conjugacy classes all have cardinality p. Hence, Γ_G^{cc} will have only one vertex that will be an isolated vertex and the rest of the $p-2$ vertices will all be connected to one another. Thus, Γ_G^{cc} is a union of one singleton graph and one complete graph of order $p-2$.

Number of edges of the conjugacy class graph of G = number of edges of $K_{p-2} = \frac{(p-2)(p-3)}{2}$.

Corollary 3.1.2. *Only one conjugacy class graph is formed with different values of r for a fixed p.*
Proof. We know that the different values of r in G gives the same conjugacy classes [19]. Since the conjugacy class graph is totally dependent on the conjugacy classes of G, hence we get the same conjugacy class graph for different values of r.

Theorem 3.1.3. *The clique number and chromatic number of Γ_G^{cc} are equal.*
Proof. Since the conjugacy class graph is simply the union of one singleton graph and one complete graph of order $p-2$, the largest induced subgraph that is a complete graph is $p-2$, and hence, $\omega(\Gamma_G^{cc}) = p-2$.

Γ_G^{cc} being the union of K_{p-2} and K_1, a minimum of $p-2$ colors will be required to color K_{p-2} and the vertex of the singleton graph K_1 can be colored with any one of the $p-2$ colors, and thus $\chi(\Gamma_G^{cc}) = p-2$.

Theorem 3.1.4. *Γ_G^{cc} is planar if $p < 7$ and non planar if $p \geq 7$.*
Proof. We know that a graph is non planar if and only if it contains a subdivision of K_5 or $K_{3,3}$ [13]. Now, from theorem 3.1.3, the clique number of Γ_G^{cc} is $p-2$. Thus, the graph is non planar if $p-2 \geq 5$, i.e. $p \geq 7$.

Now we have to check for $p < 7$, i.e. for $p = 3$ and $p = 5$.

For $p = 3$, we find that there are only two conjugacy classes and their cardinalities are coprime. Thus, the conjugacy class graph is a null graph with two vertices which is planar.

For $p = 5$, the conjugacy class graph is as given in figure 3.1 which is clearly planar. Hence the theorem.

Example 3.1.5. Let X be a K-metacyclic group of order $20 = 5(5-1)$ where the primitive root modulo r will take values 2 and 3. There are 4 conjugacy classes of X for both values of r and the conjugacy class graph is as shown in Figure 3.1:

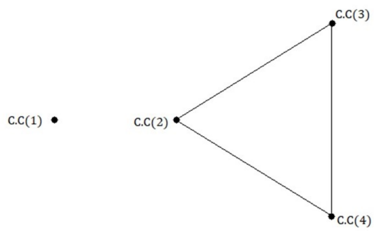

Figure 3.1

Clearly, $\omega\left(\Gamma_X^{cc}\right) = \chi\left(\Gamma_X^{cc}\right) = 3$.

Theorem 3.1.6. *The clique polynomial of* Γ_G^{cc} *is given by,* $C\left(\Gamma_G^{cc}; x\right) = x + (1 + x)^{p-2}$.

Proof. Since the conjugacy class graph is a union of complete graph of order $p-2$ and a trivial graph, the clique polynomial of Γ_G^{cc} can be computed as,

$$\begin{aligned}
C\left(\Gamma_G^{cc}; x\right) &= C(K_{p-2} \cup K_1; x) \\
&= C(K_{p-2}; x) + C(K_1; x) - 1 \\
&= (1 + x)^{p-2} + (1 + x) - 1 \\
&= x + (1 + x)^{p-2}.
\end{aligned}$$

Theorem 3.1.7. *The Independence number and the dominating number of* Γ_G^{cc} *are equal and is equal to* 2.

Proof. $\Gamma_G^{cc} = K_1 \cup K_{p-2}$. The maximum size of an independent set is 2, taking one vertex from K_{p-2} and another vertex in K_1. Hence $\alpha\left(\Gamma_G^{cc}\right) = 2$. Also, if we consider a subset taking one vertex from K_{p-2} and another vertex of K_1, we see that this subset forms a dominating set and is in fact the smallest size of a dominating set, giving the dominating number of Γ_G^{cc} to be 2. Thus, $\alpha\left(\Gamma_G^{cc}\right) = \gamma\left(\Gamma_G^{cc}\right) = 2$.

Example 3.1.8. Let X be a K-metacyclic group of order $42 = 7(7-1)$ where the primitive root modulo r will take values 3 and 5. There are 6 conjugacy classes of X for both values of r and the conjugacy class graph is as shown in Figure 3.2:

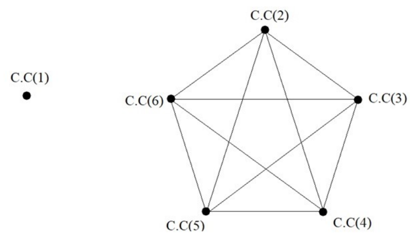

Figure 3.2

Clearly $\alpha\left(\Gamma_X^{cc}\right) = \gamma\left(\Gamma_X^{cc}\right) = 2$.

Theorem 3.1.9. *The independence polynomial of* Γ_G^{cc} *is given by* $I\left(\Gamma_G^{cc}; x\right) = 1 + (p-1)x + (p-2)x^2$.

Proof. Since $\Gamma_G^{cc} = K_1 \cup K_{p-2}$, the independence polynomial of Γ_G^{cc} can be computed as,

$$\begin{aligned} I\left(\Gamma_G^{cc}; x\right) &= I\left(K_{p-2} \cup K_1; x\right) \\ &= I\left(K_{p-2}; x\right).I\left(K_1; x\right) \\ &= (1+(p-2)x)(1+x) \\ &= 1 + x + (1+x)(p-2)x \\ &= 1 + x + (p-2)x + x^2(p-2) \\ &= 1 + (p-1)x + (p-2)x^2. \end{aligned}$$

Theorem 3.1.10. *The spectrum of* Γ_G^{cc} *for* $p \geq 5$ *is as follows:*

$$\left\{(0)^1, (-1)^{p-3}, (p-3)^1\right\}.$$

Proof. We know that the spectrum of a complete graph K_n on n vertices is given by $\{(-1)^{n-1}, (n-1)^1\}$. Also, if $X = K_{m_1} \cup K_{m_2} \cup \ldots \cup K_{m_l}$, where K_{m_i} are complete graphs on m_i vertices for $1 \leq i \leq l$, then, the spectrum of X is given by $\left\{(-1)^{\sum_{i=1}^{l} m_i - l}, (m_1-1)^1, (m_2-1)^1, ..., (m_l-1)^1\right\}$ [6].

Thus, spectrum of Γ_G^{cc}= spectrum of $K_1 \cup K_{p-2} = \{(-1)^{1+p-2-2}, (1-1)^1, (p-2-1)^1\} = \{(0)^1, (-1)^{(p-3)}, (p-3)^1\}$.

Theorem 3.1.11. *The energy of $\Gamma_G^{cc}, \varepsilon(\Gamma_G^{cc})$ is non hyperenergetic as well as non hypoenergetic.*

Proof. We know that Γ_G^{cc} is a graph with $p-1$ vertices and its spectrum is $\{0, (-1)^{p-3}, p-3\}$ for all $p \geq 5$ (where p is an odd prime).
Thus, $\varepsilon(\Gamma_G^{cc}) = 0 + p - 3 + p - 3 = 2(p-3)$.
Now, $2(n-1) = 2(p-1-1) = 2(p-2)$, and clearly $2(p-3) < 2(p-2)$.
Hence, $\varepsilon(\Gamma_G^{cc})$ is non hyperenergetic. Also,
$p \geq 5$
$2p - p \geq 6 - 1$
$2(p-3) \geq p - 1$
$\varepsilon(\Gamma_G^{cc}) \geq p - 1$.
Hence, $\varepsilon(\Gamma_G^{cc})$ is non hypoenergetic.

3.2. Conjugacy class graph of the direct product of two K-metacyclic groups.

Consider two groups G_1 and G_2. Their direct product denoted by $G_1 \times G_2$, is the set of all ordered pairs (x_1, y_1), where $x_1 \in G_1$ and $y_1 \in G_2$, with the group operation $(x_1, y_1).(x_2, y_2) = (x_1 * x_2, y_1 \Delta y_2)$, where $*$ is the group operation in G_1 and Δ is the group operation in G_2.

Two elements (x, y_1) and (x_2, y_2) are conjugate in $G_1 \times G_2$ if there exists $(z_1, z_2) \in G_1 \times G_2$ such that $(z_1, z_2)(x, y_1)(z_1^{-1}, z_2^{-1}) = (x_2, y_2)$.

Theorem 3.2.1. $\Gamma_{G\times G}^{cc}$ *is a combination of $K_{2p-4} + K_{2p-4} + K_{(p-2)^2}$ and K_3, where every vertex of K_3 is adjacent to every vertex of K_{2p-4} whose vertices are the conjugacy classes of order $p(p-1)$, but not adjacent to either of the other two complete graphs, K_{2p-4} and $K_{(p-2)^2}$ whose vertices are the conjugacy classes of order p and p^2 respectively.*

Proof. The conjugacy classes of G are as follows:
$cl(e) = \{e\}$,
$cl(x) = \{x, x^2, x^3, ..., x^{p-1}\}$,
$cl(y) = \{y, yx, yx^2, yx^3, ..., yx^{p-1}\}$,
$cl(y^2) = \{y^2, y^2x, y^2x^2, y^2x^3, ..., y^2x^{p-1}\}$,
$\vdots$
$cl(y^{p-2}) = \{y^{p-2}, y^{p-2}x, y^{p-2}x^2, y^{p-2}x^3, ..., y^{p-2}x^{p-1}\}$.
Hence, if we consider the conjugacy classes of $G \times G$, then there is 1 conjugacy class of order 1, 2 conjugacy classes of order $(p-1), (2p-4)$ conjugacy classes of order $p(p-1)$, 1 conjugacy class of order $(p-1)^2, (2p-4)$ conjugacy classes of order

p and $(p-2)^2$ conjugacy classes of order p^2.

Thus, the number of vertices of the graph $\Gamma^{cc}_{G\times G} = 2+2p-4+1+2p-4+(p-2)^2 = p^2-1$, and the graph structure is as follows:

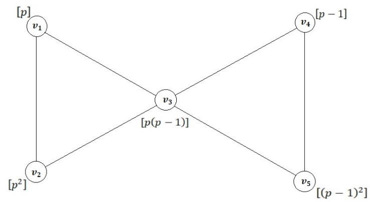

Figure 3.3

Here, v_1 represents the complete graph K_{2p-4}, where each vertex is the conjugacy class of order p,
v_2 represents the complete graph $K_{(p-2)^2}$, where each vertex is the conjugacy class of order p^2,
v_3 represents the complete graph K_{2p-4}, where each vertex is the conjugacy class of order $p(p-1)$,
v_4 represents the complete graph K_2, where each vertex is the conjugacy class of order $p-1$,
v_5 represents the complete graph K_1, where the vertex is the conjugacy class of order $(p-1)^2$.
Edges between two v_i and v_j indicates that each vertex of v_i is adjacent to each vertex of v_j in figure 3.3. v_4 and v_5 together form K_3. Hence the theorem.
Number of edges in $\Gamma^{cc}_{G\times G}$= Number of edges in K_{2p-1}+ number of edges in $K_{p^2-4}-$ number of edges in
$K_{2p-4} = \begin{pmatrix} 2p-1 \\ 2 \end{pmatrix} + \begin{pmatrix} p^2-4 \\ 2 \end{pmatrix} - \begin{pmatrix} 2p-4 \\ 2 \end{pmatrix}) = \frac{1}{2}\left(p^4-9p^2+12p+2\right).$

Example 3.2.2. For $p=3$, $\Gamma^{cc}_{G\times G}$ is given by:

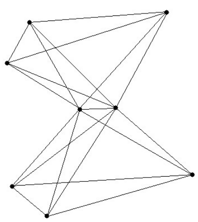

Figure 3.4

Number of vertices = 8,
Number of edges = 19.

Corollary 3.2.3. $\Gamma^{cc}_{G\times G}$ *is non planar for all values of* p.

Theorem 3.2.4. *The complement graph of* $\Gamma^{cc}_{G\times G}$ *is the union of* $2p-4$ *isolated vertices and one complete bipartite graph* K_{3,p^2-2p}.
Proof. In Figure 3.3, since each of the $2p-4$ vertices in v_3 are adjacent to all other vertices in the graph, they will be isolated vertices in $\overline{\Gamma^{cc}_{G\times G}}$. Also, all $2p-4$ vertices of v_1 will be adjacent to all vertices of v_4 and v_5, while none are adjacent to any vertex of v_2. Similarly, all $(p-2)^2$ vertices of v_2 are adjacent to all vertices of v_4 and v_5, while none are adjacent to any vertices of v_1. Hence, a complete bipartite graph K_{3,p^2-2p} is formed with 3 vertices from v_4 and v_5 and $2p-4+(p-2)^2 = p^2-2p$ vertices from v_1 and v_2.

Theorem 3.2.5. $\omega\left(\Gamma^{cc}_{G\times G}\right) = \chi\left(\Gamma^{cc}_{G\times G}\right) = p^2-4$.
Proof. The largest induced subgraph of $\Gamma^{cc}_{G\times G}$ which is a complete subgraph is $K_{2p-4} + K_{(p-2)^2} + K_{2p-4}$.
Number of vertices of this induced subgraph is p^2-4.
Hence, the clique number $\omega\left(\Gamma^{cc}_{G\times G}\right) = p^2-4$.
Now, since $K_{2p-4} + K_{(p-2)^2} + K_{2p-4}$ is a complete graph of order p^2-4, at least p^2-4 colors are required to color $\Gamma^{cc}_{G\times G}$. Since all vertices in v_4 and v_5 are not adjacent to any vertex in v_1 and v_2, the three vertices in v_4 and v_5 can be coloured with the colors chosen from v_1 or v_2. Thus, $\omega\left(\Gamma^{cc}_{G\times G}\right) = \chi\left(\Gamma^{cc}_{G\times G}\right) = p^2-4$.

Theorem 3.2.6. *The independence number of* $\Gamma^{cc}_{G\times G} = 2$.
Proof. From the structure of $\Gamma^{cc}_{G\times G}$ (Figure 3.3), the largest induced subgraph formed so that it has no edges, has exactly 2 vertices, one of the vertices being taken from either v_1 or v_2 and the other vertex taken from either v_4 or v_5. If any more vertex is added to this subgraph, the number of edges will be greater than 0. Hence, $\alpha(\Gamma^{cc}_{G\times G}) = 2$.

Theorem 3.2.7. *The dominating number of* $\Gamma^{cc}_{G\times G} = 1$.
Proof. From Figure 3.3, if we take a vertex from v_1, then we see that this vertex is adjacent to all other vertices in $\Gamma^{cc}_{G\times G}$. Hence, $\gamma(\Gamma^{cc}_{G\times G}) = 1$.

3.2. Line graph and complement graph of the conjugacy class graph of K-metacyclic group.

A line graph $L(G)$ of a graph G is a graph where the vertex set consists of lines of G and two lines of G are adjacent in $L(G)$ if and only if they are incident in G. Also, the complement graph G of a graph G is a graph which has the same vertex set as G and two vertices are adjacent in G if and only if they are not adjacent in the original graph. Both these graphs serve as a valuable tool for studying and analysing structural properties of graphs, offering insights into connectivity, cliques, and other graph-theoretic characteristics. This subsection is dedicated to the examination of the line graph and complement graph arising from the conjugacy class graph of a K-metacyclic group, wherein specific properties of these resultant graphs are investigated as well.

Theorem 3.3.1. *The line graph of* $\Gamma^{cc}_{G\times G}$ *is a 2(p-4)-regular graph of order* $\binom{p-2}{2}$.

Proof. The line graph of Γ^{cc}_{G} will simply be formed from the complete graph, ignoring the singleton graph, and hence will be a regular graph.
Number of vertices in the complete graph $=p-2$.
Number of edges $= \binom{p-2}{2}$.
Thus, number of vertices in the line graph $=\binom{p-2}{2}$.
Also, degree of each vertex in the complete graph $= p-3$.
Thus, degree of each vertex in the line graph $= 2(p-3)-2 = 2(p-4)$.
Hence the theorem.

Example 3.3.2. For $p = 7$, line graph of Γ^{cc}_{G}. is as follows:

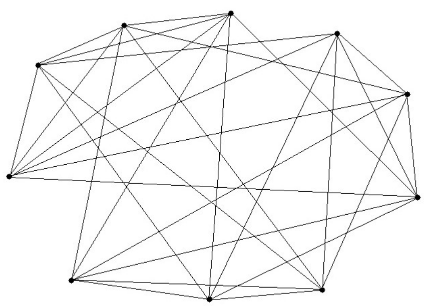

Figure 3.5

Theorem 3.3.3. *The chromatic number of the line graph of* Γ^{cc}_{G}. *is bounded and,*

$$3 \leq \chi(L(\Gamma^{cc}_{G}) \leq 2p-7.$$

Proof. The upper bound for the chromatic number of any graph is one more than the maximum degree of any vertex in the graph. Since the maximum degree in

$(L(\Gamma_G^{cc})$ is $2p-8$, hence the upper bound is $2p-7$. Also, since every edge in Γ_G^{cc} is incident to exactly two vertices and every pair of edges share exactly one common endpoint, it follows that the line graph of Γ_G^{cc} will contain at least one triangle. Hence, the lower bound of the chromatic number of $L(\Gamma_G^{cc})$ is 3.

Theorem 3.3.4. *The complement graph of* $\Gamma_G^{cc}, \overline{\Gamma_G^{cc}}$ *is a star graph of order* $p-1$.
Proof. In the complement graph of Γ_G^{cc} the isolated vertex becomes the vertex with degree $p-2$ and all other vertices contained in the complete graph becomes vertices of degree 1, which forms a star graph. Since the number of vertices in the complement graph remain the same as the conjugacy class graph, hence, we get the star graph of order $p-1$.

Example 3.3.5. For p=7, complement graph of Γ_G^{cc} is as follows:

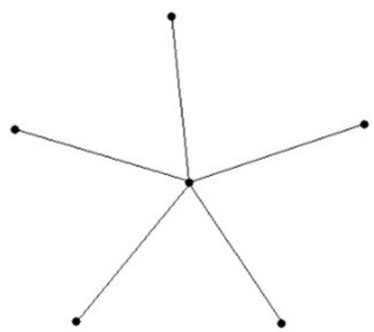

Figure 3.6

Corollary 3.3.6.
1. $\overline{\Gamma_G^{cc}}$ *is a perfect graph, since* $\chi(\overline{\Gamma_G^{cc}}) = \omega(\overline{\Gamma_G^{cc}}) = 2$.
2. The independence number of $\overline{\Gamma_G^{cc}}$ *is* $p-2$.
3. The dominating number of $\overline{\Gamma_G^{cc}}$ *is* 1.

Acknowledgement

The first author is thankful for the financial support provided by Junior Research Fellowship-University Grants Commission under Ministry of Human Resource Development. F.No.16-6 (Dec. 2018)/ 2019(NET/ CSIR), UGC-Ref. No.: 1102/(CSIR-UGC NET DEC.2018).

References

[1] Armstrong M. A., Groups and symmetry, Springer-Verlag, New York Berlin Heidelberg, London Paris Tokyo Hong kong, 1987.

[2] Bertram E. A., M.Herzog and A.Mann, On a graph related to conjugacy classes of groups, Bull. London Math. Soc., 22(6) (1990), 569-575.

[3] Bianchi M., Herzog M., Pacifici E., Saffirio G., On the regularity of a graph related to conjugacy classes of groups, European Journal of Combinatorics, 33 (2012), 1402–1407.

[4] Chartrand G., Introductory Graph theory, Dover publications, Inc. New York, 1985.

[5] Childs L. N., A concrete introduction to higher algebra, 2nd edition, Springer, 1995.

[6] Dutta J. and Nath R. K., Spectrum of commuting graphs of some classes of finite groups, Matematika, Volume 33, No. 1 (2017), 87-95.

[7] Godsil C., Gordon R., Algebraic graph theory, 1st edition, Springer New York, 2001.

[8] Hall M., The theory of groups, Chelsea Publishing Company, New York, 1959.

[9] Herstein I. N., Topics in Algebra, 2nd edition, Wiley Student edition, 1964.

[10] Hoede C. and Li X., Clique polynomials and independent set polynomials of graphs, Discrete Mathematics, 125 (1994), 219-228.

[11] Johanson D. L., Topics in the theory of group presentations, London Mathematical society Lecture note series, 42, Cambridge university press, 1980.

[12] Khoshnevis D. and Mostaghim Z., Some properties of Graph Related to Conjugacy Classes of Special Linear Group $SL_2(F)$, Mathematical Sciences Letters, 4, No. 2 (2015), 153-156.

[13] Koltz W., A constructive proof of Kuratowski's theorem, https://www.researchgate.net/publication/256078009, (1989).

[14] Li X., Shi Y., Gutman I., Graph Energy, Springer, New York, 2012.

[15] Moradipour K., Sarmin N. H., Erfanian A., On Graphs Associated to Conjugacy Classes of Some Metacyclic 2-Groups, Journal of Basic and Applied Scientific Research, (2013).

[16] Nabilah Najmuddin N. Y., Sarmin N. H., Erfanian A., Rahmat H., The independence and clique polynomial of the conjugacy class graph of dihedral group, Malaysian Journal of Fundamental and Applied Sciences, MJFAS, (2018).

[17] Patra K., PhD thesis submitted to the Department of Mathematics, Gauhati University, 1984.

[18] Sarmin N. H., Bilhikmah N. H., Mohamed Saleh S. and Mohd Noor Alia H. B., The Conjugacy Classes, Conjugate Graph and Conjugacy Class Graph of Some Finite Metacyclic 2-Groups, Menemui Matematik (Discovering Mathematics), Vol. 38, No. 1 (2016), 1-12.

[19] Yangertola, Patra K., Conjugacy classes and conjugate graph of a K-metacyclic group, International journal of Advanced and Applied Mathematics, Volume 10, Issue 11 (2023), 176-183.

[20] Zulkarnain A., Sarmin Nor H., Hassim H. I. Mat, The conjugacy class graphs of non-abelian 3-groups, Malaysian Journal of Fundamental and Applied Sciences, MJFAS, (2020).